

## Production Cross-sections for Unstable Particles

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### Abstract

The top-quark,  $W$  and  $Z^0$  bosons have widths that are a sizable fraction of their masses and will be produced copiously at upcoming accelerators. Yet  $S$ -matrix theory cannot treat unstable particles as external states. Dealing with complete matrix elements involving their decay products complicates calculations considerably and is unnecessary in many practical situations. It is shown how to construct physically meaningful production cross-sections for unstable particles by extracting that part of the matrix element that corresponds to finite-range space-time propagation. This procedure avoids the need to define unstable particles in external states and we argue its favour as providing a solution to a long-standing problem in physics. As an example the results are applied to the calculation of the cross-section  $\sigma(e^+e^- \rightarrow Z^0 Z^0)$ .

# 1 Introduction

The treatment of unstable particles in Quantum Field Theory is fraught with difficulties. The problems arise from the fact that the particles cannot be represented as asymptotic states because they decay a finite distance from the interaction region. Veltman [2] showed that the  $S$ -matrix satisfies unitarity and causality on the Hilbert space of stable particle states even when unstable particles appeared in intermediate states. Many unstable particles are so short-lived that they cannot be observed directly and only their decay products are seen. In such cases the treatment of processes involving unstable particles in terms of their stable decay products is an accurate description of the physics. It does however substantially increase the complexity of a calculation as the number of external particles that must be dealt with is larger. Treating unstable particles in this way also begs the question of their existence. Muons, kaons,  $B$ -mesons and neutrons are observed directly and can be manipulated experimentally before they decay and thus considering them purely in terms of their decay products is clearly inadequate. Moreover there is no fundamental difference between these particles and their shorter-lived cousins. The difference is merely one of experimental resolution.

The problem of the unstable particle was first formulated by Peierls [1] in the early 1950's and is particularly relevant at the present time with the upcoming commissioning of LEP200. At this machine electrons and positrons are collided with sufficient energy to produce  $W^+W^-$  and  $Z^0Z^0$  pairs. Experimental precision is such that electroweak radiative corrections are required in the comparison of theory with experiment. However the  $W$  and  $Z^0$  have a finite width that is approximately 3% of their mass and cannot, by any means, be considered stable. To obtain the required theoretical accuracy the complete production and decay process should be considered. This is a process that contains six external particles and the calculation of the complete electroweak radiative corrections is daunting indeed. It is certainly very much more complicated than for the already taxing corrections to  $e^+e^- \rightarrow W^+W^-$  [3, 4, 5, 6] with only four. In addition, four fermion final states are produced in  $e^+e^-$  collisions via mechanisms other than intermediate  $W$  or  $Z^0$  production. All such processes should be calculated and included for a consistent calculation.

Width effects have also been considered as a possible mechanism for generating large  $CP$ -violating effects in top quark decays [7, 8].

In most situations to date the finite lifetime of the unstable particle is either ignored or allowed for by what amounts to a convolution of the stable particle matrix element with a Breit-Wigner distribution [9, 10]. The relative merits of a variety of expansion techniques, that had previously been used in other physical situations, to treat finite width effects has also been examined [11, 12].

A number of problems exist with previous analyses. The authors of ref.[11] apparently misinterpret the method described in ref.s[13, 14] as being an expansion of the matrix element about the real renormalized mass followed by the addition, by hand,

of an imaginary part to that mass. In fact the method involves Laurent expansion of the complete matrix element about its complex pole followed by the perturbative expansion of the pole position, residue and background, so generated, about the renormalized mass. The error is repeated in ref.[12] where it leads to the appearance of so-called ‘threshold singularities’ and/or complex scattering angles when one is below production threshold. This feature is clearly a calculational artifact. The branch point corresponding to the production of a pair of unstable  $W$  bosons, as considered by these authors, lies away from the real  $s$ -axis. One therefore never encounters the branch point as the centre-of-mass energy changes and one is neither strictly above nor below threshold. The apparent problem of threshold singularities arises when one begins an expansion using the real renormalized mass which places the threshold branch point on the real axis.

The question, “What is the production cross-section for  $Z^0 Z^0$  in  $e^+e^-$  annihilation?”, should be a physically well-posed one. One need only perform a *gedanken* weakening of the Standard Model couplings of the  $Z^0$ ’s in order to make them sufficiently long-lived that their existence could be confirmed by, say, vertex detectors. Above their production threshold, physical  $Z^0$  pairs would be expected to be the dominant source of  $(f_1 \bar{f}_1)(f_2 \bar{f}_2)$  final states because of their resonant enhancement and thus a definitive answer would avoid the need to consider processes with six external particles. Yet  $S$ -matrix theory provides no unambiguous response.

Note in passing that for the inverse process, the question as to what is the lifetime of an unstable particle is not well-posed. It has been known for a long time [15] that this depends on the manner in which the particle was prepared. It is, for example, meaningful to inquire as to the lifetime of an unstable particle with a definite 4-momentum.

Many years ago, a number of authors [16, 17] used residues at poles in the  $S$ -matrix element for processes containing intermediate unstable particles to define  $S$ -matrix elements for external unstable particles. Although these generalized  $S$ -matrix elements possess intriguing mathematical properties, such as generalized unitarity relations, their physical meaning is less than clear and it is not obvious how they may be used to calculate measurable production cross-sections.

In this paper an answer will be given to the question posed above. Although standard perturbation theory cannot tolerate unstable particles as asymptotic states, it will be shown that it is nevertheless possible to formulate a physically motivated procedure to consistently extract from the  $S$ -matrix element for  $e^+e^- \rightarrow (f_1 \bar{f}_1)(f_2 \bar{f}_2)$  that part which proceeds via the production of an intermediate  $Z^0$  pair and their subsequent decay. Summation over all such final states then gives the  $Z^0$  production cross-section. In this procedure standard perturbation theory is employed. We do not attempt to define an effective propagator for an unstable particle or modify the usual Feynman rules in any way. Nor is an  $S$ -matrix element defined with external unstable particles but nevertheless a meaningful physical cross-section is obtained. The uniqueness of the procedure is discussed at the end of section 2.

An immediate consequence of this is that the above reaction may be separated into four distinct physical, and consequently separately gauge-invariant, processes

$$e^+e^- \longrightarrow \left\{ \begin{array}{l} Z^0 \rightarrow f_1\bar{f}_1 \\ Z^0 \rightarrow f_2\bar{f}_2 \end{array} \right.$$

$$e^+e^- \longrightarrow \left\{ \begin{array}{l} Z^0 \rightarrow f_1\bar{f}_1 \\ f_2\bar{f}_2 \end{array} \right.$$

$$e^+e^- \longrightarrow \left\{ \begin{array}{l} f_1\bar{f}_1 \\ Z^0 \rightarrow f_2\bar{f}_2 \end{array} \right.$$

$$e^+e^- \longrightarrow \left\{ \begin{array}{l} f_1\bar{f}_1 \\ f_2\bar{f}_2 \end{array} \right.$$

The first of these is the one we will consider here. It is the dominant mode in a variety of kinematic regions of experimental interest. The others may be calculated by similar methods if the experimental situation requires it.

The calculation of  $\sigma(e^+e^- \rightarrow Z^0 Z^0)$  is technically much more demanding than for the process  $e^+e^- \rightarrow Z^0 \rightarrow f\bar{f}$ . In the latter case the  $Z^0$  is produced with a fixed invariant mass and but for the former the invariant masses of the  $Z^0$ 's vary over some kinematically allowed region and must somehow be incorporated into phase-space integrations. As discussed above, naïve generalizations of the methods of ref.s[13, 14] soon encounter difficulties or apparent arbitrariness in how to proceed. It will be shown that, by carefully respecting the Lorentz-structure of the analytic  $S$ -matrix at each stage, a unique procedure is indicated with no flexibility in the final result.

A number of authors [18, 19, 20] have studied the effects of interactions between the final-state decay products of unstable particles. These are generally found to be suppressed and the physical reason for this will be given at the end of the paper.

## 2 Identification of an Unstable Particle

We will begin by examining the form and properties of the dressed propagator or Green's function for an unstable particle. For simplicity a scalar particle will be considered but the generalization to fermions or vector bosons is straightforward. Although the dressed propagator does not by itself represent a physical observable we will use it to investigate what properties of the analytic  $S$ -matrix element may be employed to indicate the presence of an unstable particle.

The equation of motion of a free scalar particle are given by the Klein-Gordon equation

$$(\square + m^2)\phi(x) = 0. \tag{1}$$

The space-time evolution of the scalar field is governed by the propagator  $\Delta(x' - x)$  which in coordinate space may be represented as

$$\Delta(x' - x) = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-ik \cdot (x' - x)}}{k^2 - m^2 + i\epsilon}. \quad (2)$$

When interactions are switched on the dressed propagator

$$\Delta(x' - x) = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-ik \cdot (x' - x)}}{k^2 - m^2 - \Pi(k^2) + i\epsilon} \quad (3)$$

is generated. Here  $\Pi(k^2)$  is scalar's self-energy. For reasons that will become apparent soon the integrand in will be split into three pieces. Thus

$$\begin{aligned} \Delta(x' - x) &= \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x' - x)} \\ &\times \left[ \frac{1}{2k_0} \cdot \frac{F(s_p)}{k_0 - \sqrt{\vec{k}^2 + s_p}} + \frac{F(k^2) - F(s_p)}{k^2 - s_p} + \frac{1}{2k_0} \cdot \frac{F(s_p)}{k_0 + \sqrt{\vec{k}^2 + s_p}} \right]. \quad (4) \end{aligned}$$

The quantity  $s_p$  is the position of the complex pole of the dressed propagator. It is a solution to the equation

$$s - m^2 + \Pi(s) = 0 \quad (5)$$

Also

$$F(s) = \frac{s - s_p}{s - m^2 + \Pi(s)} \quad (6)$$

from which it follows by l'Hôpital's rule  $F(s_p) = (1 + \Pi'(s_p))^{-1}$ .

Performing the integration in  $k_0$  on the first and third terms in (4) for  $t \neq t'$  gives

$$\begin{aligned} \Delta(x' - x) &= -i \int \frac{d^3 k}{(2\pi)^3 2k_0} e^{-ik \cdot (x' - x)} \theta(t' - t) F(s_p) \\ &+ \int \frac{d^4 k}{(2\pi)^4} \frac{F(k^2) - F(s_p)}{k^2 - s_p} \\ &- i \int \frac{d^3 k}{(2\pi)^3 2k_0} e^{ik \cdot (x' - x)} \theta(t - t') F(s_p). \quad (7) \end{aligned}$$

In eq.(7)  $k_0 = \sqrt{\vec{k}^2 + s_p}$ . The second term on the right-hand side of (4) has no poles in  $k_0$  near the physical region and is identifiable as a contact interaction locally.

The first terms of (4) and (7) describe the propagation of the scalar field forward in time. The last term is usually thought of as the forward propagation of the anti-particle. It is tempting to conclude that the first term describes the motion of the physical unstable particle. Indeed terms of rather similar form are obtained when Bogoliubov causality is applied to the dressed propagator [21]. The problem with

the decomposition in (4) is that the first and last terms are not separately Lorentz invariant. One would expect that all observers should agree on whether an unstable particle has been created. In processes in which the propagator is connected to external stable particles, one or other of the particle or antiparticle pieces will be energetically disfavoured and represent a virtual state that is protected from observation by the uncertainty principle. Both pieces must be taken together to obtain a Lorentz-invariant expression. The propagator (4) may then be written

$$\Delta(x' - x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik \cdot (x' - x)} \left[ \frac{F(s_p)}{k^2 - s_p} + \frac{F(k^2) - F(s_p)}{k^2 - s_p} \right]. \quad (8)$$

The first term is the finite-range piece and the second is a contact term.

In the following we will identify the unstable particle by its finite-range propagation within a physical matrix element which may be obtained by procedures similar to those applied above to the dressed propagator. This characterization of an unstable particle avoids the need to define them in external states and is therefore amenable to  $S$ -matrix theory. It is as equally applicable to the directly-observable neutron as to the  $Z^0$ .

We will be interested in calculating production cross-sections for unstable particles and will therefore drop contact terms since they describe the prompt production of the final state without the intermediate production of an unstable particle. The contact term is normally subdominant and in many practical situations lies below experimental error. In such cases it may also be dropped and the production cross-section for the unstable particle provides an adequate description of the physics. Where the contact term is required, as for example in LEP1 physics, it may be added without difficulty [13].

It is interesting to note that the finite-range and contact interaction are both produced by the same field yet are physically distinguishable. There is therefore not a one-to-one correspondence between the field and the particle.

As stated above we have chosen to tag the existence of an unstable particle by its finite space-time propagation which involves decomposition of an  $S$ -matrix element into two distinguishable pieces. It may be one could use some other property to identify the presence of an unstable particle and divide up the matrix element accordingly but such an alternative would necessarily mean mixing propagating and non-propagating modes of the field in which case the physical interpretation becomes unclear. Identifying the unstable particle by its finite space-time propagation seems, however, to be the choice which is most physically and intuitively reasonable. It is, after all, this property that allows  $b$ -quarks to be identified by vertex detectors.

### 3 The cross-section for $e^+e^- \rightarrow Z^0 Z^0$

The above considerations allow the cross-section for  $e^+e^- \rightarrow Z^0 Z^0$  to be calculated. We start from the full matrix element for  $e^+e^- \rightarrow (f_1 \bar{f}_1)(f_2 \bar{f}_2)$ . For the present

purposes it will be assumed that the fermions are massless.

The calculation of the production cross-section for  $Z^0$  pairs proceeds along classical lines using standard Feynman rules. The squared invariant masses of the  $f_1\bar{f}_1$  and  $f_2\bar{f}_2$  pairs are  $p_1^2$  and  $p_2^2$ . The matrix element that accounts for physical  $Z^0 Z^0$  production is obtained by extracting the part that is resonant in both  $p_1^2$  and  $p_2^2$ . The cross-section is then obtained by squaring and summing over all possible final states.

The part of the matrix element that is resonant in  $p_1^2$  but not  $p_2^2$  describes the production of a physical  $Z^0$  and a pair of fermions  $f_2\bar{f}_2$ . This piece must be combined with other Feynman diagrams yielding the same final state but no additional intermediate  $Z^0$  in order to produce a gauge-invariant result.

The part of the matrix element that is resonant in  $p_2^2$  describes  $Z^0$  production in association with the fermions  $f_1\bar{f}_1$  and the part that is resonant in neither variable describes direct four-fermion production. Thus the complete matrix element  $Z^0$  separates into distinct pieces corresponding to the four distinct physical processes given previously.

The part of the full matrix element that can give rise to doubly resonant contributions can be written as

$$\begin{aligned} \mathcal{M} = & \sum_i [\bar{v}_{e+} T_{\mu\nu}^i u_{e-}] M_i(t, u, p_1^2, p_2^2) \\ & \times \frac{1}{p_1^2 - M_Z^2 - \Pi_{ZZ}(p_1^2)} [\bar{u}_{f_1} \gamma^\mu (V_{Zf_L}(p_1^2) \gamma_L + V_{Zf_R}(p_1^2) \gamma_R) v_{\bar{f}_1}] \\ & \times \frac{1}{p_2^2 - M_Z^2 - \Pi_{ZZ}(p_2^2)} [\bar{u}_{f_2} \gamma^\nu (V_{Zf_L}(p_2^2) \gamma_L + V_{Zf_R}(p_2^2) \gamma_R) v_{\bar{f}_2}] \end{aligned} \quad (9)$$

where  $T_{\mu\nu}^i$  are Lorentz covariant tensors that span the tensor structure of the matrix element and  $\gamma_L$ ,  $\gamma_R$  are the usual helicity projection operators. The  $M_i$ ,  $\Pi_{ZZ}$  and  $V_{Zf}$  are Lorentz scalars that are analytic functions of the independent Lorentz invariants of the problem. It is always possible to separate the matrix element into so-called ‘standard’ Lorentz covariants and Lorentz scalar functions [22, 23, 24, 25]. There is a limited amount of flexibility as to where this separation is made. One must not, for example, introduce spurious kinematic singularities and hence the final results for the matrix elements and cross-sections obtained in what follows are not affected by this freedom. It is sometimes helpful, although not essential, that the  $T_{\mu\nu}^i$  be constructed to be separately gauge-invariant [26]. As before the external fermion wave-functions are denoted  $u$  and  $v$  with appropriate subscripts. The  $Z$ - $\gamma$  mixing has been neglected but it will not alter the structure of the final result [14].

To extract the piece of the matrix element that corresponds to finite propagation of both  $Z^0$ ’s we extract the leading term in a Laurent expansion in  $p_1^2$  and  $p_2^2$  of the analytic Lorentz-invariant part of eq.(9) leaving the Lorentz-covariant part untouched. This is the doubly-resonant term and is given by

$$\mathcal{M} = \sum_i [\bar{v}_{e+} T_{\mu\nu}^i u_{e-}] M_i(t, u, s_p, s_p)$$

$$\begin{aligned}
& \times \frac{F_{ZZ}(s_p)}{p_1^2 - s_p} [\bar{u}_{f_1} \gamma^\mu (V_{Zf_L}(s_p) \gamma_L + V_{Zf_R}(s_p) \gamma_R) v_{\bar{f}_1}] \\
& \times \frac{F_{ZZ}(s_p)}{p_2^2 - s_p} [\bar{u}_{f_2} \gamma^\nu (V_{Zf_L}(s_p) \gamma_L + V_{Zf_R}(s_p) \gamma_R) v_{\bar{f}_2}]
\end{aligned} \tag{10}$$

where  $F_{ZZ}$  defined by a relation like (6). It should be emphasized that eq.(10) the exact form of the doubly-resonant matrix element to all orders in perturbation theory that we will now specialize to leading order. It is free of threshold singularities noted that were found by other authors [12]. Although  $M_i$  was obtained directly by means of eq.(9) the general form of eq.(10) with its factorization between initial and final states is guaranteed by Fredholm theory. Note that the external wave-functions do not figure in the expansion as they are external sources and sinks and do not contribute to defining the finite-range part. The kinematic factors associated with external particles are thus unchanged by the expansion so that squaring of the matrix element and phase space integrals can be performed without added difficulty. Moreover since the doubly-resonant matrix element is independent of the singly- and non-resonant parts of the complete matrix element, (10), enjoys the same invariance properties, such as gauge-independence, as the complete matrix element.

A word is in order here about momentum conservation within the Lorentz scalar functions such, as  $M_i$ , during analytic continuation. The analytic continuation of the  $M_i$  to complex values of  $p_1^2$  and  $p_2^2$  is a well-defined procedure that formed the basis for defining generalized  $S$ -matrix elements for external unstable particles and about which an extensive literature exists [16, 17]. The  $S$ -matrix requires that the external massless fermions stay on-shell and conservation of 4-momentum implies that  $\mathcal{M}$  can be a function of at most four independent Lorentz scalars [27, section 4.3]. These we are free to take as  $t$ ,  $u$ ,  $p_1^2$  and  $p_2^2$ . Any of these variables may be continued without upsetting the constraints of momentum conservation. Thus for example, the well-known condition  $s + t + u = p_1^2 + p_2^2$  remains satisfied as  $p_1^2$  and  $p_2^2$  are continued away from the real axis by compensating changes in the dependent variable  $s$ .

In lowest order eq.(10) becomes, up to overall multiplicative factors,

$$\begin{aligned}
\mathcal{M} &= \sum_{i=1}^2 [\bar{v}_{e^+} T_{\mu\nu}^i u_{e^-}] M_i \\
&\times \frac{1}{p_1^2 - s_p} [\bar{u}_{f_1} \gamma^\mu (V_{Zf_L} \gamma_L + V_{Zf_R} \gamma_R) v_{\bar{f}_1}] \\
&\times \frac{1}{p_2^2 - s_p} [\bar{u}_{f_2} \gamma^\nu (V_{Zf_L} \gamma_L + V_{Zf_R} \gamma_R) v_{\bar{f}_2}].
\end{aligned} \tag{11}$$

where  $T_{\mu\nu}^1 = \gamma_\mu (\not{p}_{e^-} - \not{p}_1) \gamma_\nu$ ,  $M_1 = t^{-1}$ ;  $T_{\mu\nu}^2 = \gamma_\nu (\not{p}_{e^-} - \not{p}_2) \gamma_\mu$ ,  $M_2 = u^{-1}$  and the final state vertex corrections take the form  $V_{Zf_L} = ie\beta_L^f \gamma_L$  and  $V_{Zf_R} = ie\beta_R^f \gamma_R$ . The left- and right-handed couplings of the  $Z^0$  to a fermion  $f$  are

$$\beta_L^f = \frac{t_3^f - \sin^2 \theta_W Q^f}{\sin \theta_W \cos \theta_W}, \quad \beta_R^f = -\frac{\sin \theta_W Q^f}{\cos \theta_W}.$$



Squaring the matrix element and integrating over the final state momenta for fixed  $p_1^2$  and  $p_2^2$  gives

$$\frac{\partial^3 \sigma}{\partial t \partial p_1^2 \partial p_2^2} = \frac{\pi \alpha^2}{s^2} (|\beta_L^e|^4 + |\beta_R^e|^4) \left\{ \frac{t}{u} + \frac{u}{t} + \frac{2(p_1^2 + p_2^2)^2}{ut} - p_1^2 p_2^2 \left( \frac{1}{t^2} + \frac{1}{u^2} \right) \right\} \rho(p_1^2) \rho(p_2^2) \quad (12)$$

with

$$\begin{aligned} \rho(p^2) &= \frac{\alpha}{6\pi} \sum_f (|\beta_L^f|^2 + |\beta_R^f|^2) \frac{p^2}{|p^2 - s_p|^2} \theta(p_0) \theta(p^2) \\ &\approx \frac{1}{\pi} \cdot \frac{p^2 (\Gamma_Z/M_Z)}{(p^2 - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \theta(p_0) \theta(p^2) \end{aligned}$$

Note that  $\rho(p^2) \rightarrow \delta(p^2 - M_Z^2) \theta(p_0)$  as  $\text{Im}(s_p) \rightarrow 0$  which is the result obtained by cutting a free propagator. The variables  $s$ ,  $t$ ,  $u$ ,  $p_1^2$  and  $p_2^2$  in eq.(12) arise from products of standard covariants and external wave functions and therefore take real values dictated by the kinematics.

Integrating over  $t$ ,  $p_1^2$  and  $p_2^2$  leads to

$$\sigma(s) = \int_0^s dp_1^2 \int_0^{(\sqrt{s} - \sqrt{p_1^2})^2} dp_2^2 \sigma(s; p_1^2, p_2^2) \rho(p_1^2) \rho(p_2^2), \quad (13)$$

where

$$\sigma(s; p_1^2, p_2^2) = \frac{2\pi\alpha^2}{s^2} (|\beta_L^e|^4 + |\beta_R^e|^4) \left\{ \left( \frac{1 + (p_1^2 + p_2^2)^2/s^2}{1 - (p_1^2 + p_2^2)/s} \right) \ln \left( \frac{-s + p_1^2 + p_2^2 + \lambda}{-s + p_1^2 + p_2^2 - \lambda} \right) - \frac{\lambda}{s} \right\}$$

and  $\lambda = \sqrt{s^2 + p_1^4 + p_2^4 - 2sp_1^2 - 2sp_2^2 - 2p_1^2 p_2^2}$ . For  $p_1^2 = p_2^2 = M_Z^2$  this agrees with known results [28].

The form of eq.(13) superficially rather similar to expressions obtained by others [9, 10]. These authors apply their method to tree-level matrix elements to obtain a cross-section in the form of a gauge-invariant convolution integral. Applying the same method to the one-loop or higher matrix-elements would require the evaluation the form factors,  $M_i$ , cross-section in the integrand with off-shell momenta,  $p_1^2$  and  $p_2^2$ , and will thus generate a gauge-dependent result. Here the extraction of the full doubly resonant part from the matrix-element generates a gauge-invariant result at all orders. In higher orders the integrations in  $p_1^2$  and  $p_2^2$  are relatively simple as they only need to be performed over kinematically generated factors and not over complete higher order form-factors. The price is that the form-factors must be evaluated for complex arguments.

## 4 Discussion

In the recent past attempts have been made to produce gauge-invariant self-energies, vertex corrections and box diagrams in order to ease problems of spurious gauge-dependence that can often arise in certain types of calculations. This is done by

extracting ‘universal’ pieces from vertex and box diagrams and combining with self-energies. The universal pieces have been obtained by a variety of methods [29, 30, 31] such as the so-called pinch technique. These self-energies may be resummed in the usual way along with their universal pieces. While there is no doubt that the self-energies do not display explicit gauge parameter dependence, their use in resummations is highly questionable as there are no Feynman diagrams that would generate such a resummation of vertex pieces. Adding universal pieces to the self-energies in this way also runs the risk, as was pointed out in ref.[14], of shifting the position of the complex pole of the  $S$ -matrix element, that is governed solely by the self-energy. The residue at the pole is also at risk of being altered by these procedures. Although there are indications that this does not happen at one loop, the pinch technique provides no firm proof that this feature persists in higher orders.

In ref.s[13, 14] it was shown how to obtain an exactly gauge-invariant perturbation expansion to all orders of the matrix element for the process  $e^+e^- \rightarrow f\bar{f}$  near the  $Z^0$  resonance. This involved performing a Laurent expansion of the matrix element about the pole and thereby extracting the resonant and non-resonant background terms. Gauge invariance of the non-resonant background is achieved by a combination of non-resonant Feynman diagrams and non-resonant contributions coming from the dressed self energy. In the alternative approaches discussed above, quite the opposite is true. There pieces from vertices and boxes are migrated into the self-energy. Yet the split between resonant and non-resonant terms is not just a mathematical trick. As seen in the foregoing, resonant and non-resonant parts describe distinct physical processes.

Recently the background field method has been used to generate gauge-invariant self-energies [32]. This constitutes a genuine self-consistent perturbation expansion that can, in principle, be extended to all orders. Resummations can be performed without the risk of shifting the pole position. Nevertheless, the resulting matrix elements should be subjected to Laurent expansion in order to make a clean separation between distinct physical processes that is a feature of the exact  $S$ -matrix element.

A number of groups [18, 19, 20] have examined the production of charged unstable particles;  $W^+W^-$ ,  $t\bar{t}$  etc. Many such effects are suppressed because the unstable particle propagates a finite distance from the production point before producing the final-state decay products.

The phenomenon of suppression of corrections in the presence of resonant states has already been observed for the case of the  $Z^0$ -boson [33, 34]. In an exact calculation of the QED initial-final state interference on the resonant part of the matrix element for  $e^+e^- \rightarrow f\bar{f}$ . It was found that the corrections pass through zero near  $s = M_Z^2$  which may be understood as the  $Z^0$  propagating a finite distance from its production point before decaying.

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